

# Recovering modified Newtonian dynamics by changing inertia

Lingjun Wang<sup>\*</sup>

20/F, Building 128, Nanhuxiyuan, Chaoyang District, Beijing, China

## ABSTRACT

Milgrom’s modified Newtonian dynamics (MOND) has done a great job on accounting for the rotation curves of a variety of galaxies by assuming that Newtonian dynamics breaks down for low acceleration typically found in the galactic contexts. This breakdown of Newtonian dynamics may be a result of modified gravity or a manifest of modified inertia. The MOND phenomena are derived here based on three general assumptions: 1) Gravitational mass is conserved; 2) Inverse-square law is applicable at large distance; 3) Inertial mass depends on external gravitational fields.

These assumptions not only recover the deep-MOND behaviour, the accelerating expansion of the universe is also a result of these assumptions. Then Lagrangian formulae are developed and it is found that the assumed universal acceleration constant  $a_0$  is actually slowly varying by a factor no more than 4. This varying ‘constant’ is just enough to account for the mass-discrepancy presented in bright clusters.

**Key words:** gravitation – dark matter – cosmology: theory.

## 1 INTRODUCTION

The modified Newtonian dynamics (MOND), originally proposed by Milgrom (1983) as an alternative to the cold dark matter paradigm to account for the rotation curves of spiral galaxies, has extended its success to dwarfs, low surface brightness galaxies (LSB) and ellipticals (see Sanders & McGaugh 2002, for a review). When confronting with clusters, especially rich clusters, MOND shows some drawback. On the cluster scale, MOND still needs dark matter, which is what MOND was particularly devised to eliminate. To overcome this difficulty, neutrinos were speculated to be responsible (Sanders 2003; Angus et al. 2007; Gentile, Zhao & Famaey 2008). Neutrinos with mass  $\sim 2$  eV, marginally allowed by current most accurate neutrino mass measurement, contributing negligibly to galaxies’ mass budget, could be dynamically significant in clusters of galaxies. Though this hypothesis is successful in some aspects, it is still controversial (e.g. Pointecouteau & Silk 2005; Angus, Famaey & Buote 2008).

Despite this drawback, MOND has drawn much attention because of its impressive success compared with the standard cold dark matter paradigm, which is facing with some difficulties, especially on the galactic scale. MOND is a phenomenological theory that may be interpreted in different ways. First of all, it may indicate a breakdown of Newtonian gravity (Bekenstein & Milgrom 1984) where the standard Poisson equation is replaced by  $\nabla \cdot [\mu(|\nabla\varphi|/a_0)\nabla\varphi] = 4\pi G\rho$ , and  $a_0$  ( $\sim 1.2 \times 10^{-8} \text{ cm s}^{-2}$ ), introduced by MOND, is a new acceleration constant below which dynamics and/or gravity become significantly non-Newtonian. Milgrom (2002) reviewed this interpretation which relates the gravity to a potential flow. Although this is a field need more investigation, the lack of profound physical foundation makes this interpretation less attractive. A second interpretation is that gravitational constant increases when accelerations are lower than  $a_0$  (Bekenstein 2004). This relativistic extension of MOND can mimic MOND’s behaviour at low acceleration extreme, but it is still a subject of debate (e.g. Reyes et al. 2010).

A third interpretation is what Milgrom (1983) proposed that the Newtonian dynamics may break down at low accelerations. Instead of the usual  $\mathbf{F} = m\mathbf{a}$ , Milgrom (1983) suggested a modified dynamics

$$\mathbf{F} = m_g \mu(a/a_0) \mathbf{a}, \quad (1)$$

$$\mu(x \gg 1) \approx 1, \quad \mu(x \ll 1) \approx x,$$

<sup>\*</sup> E-mail: initapp@hotmail.com

where  $m_g$  is the gravitational mass of the body moving in the field. This relation is equivalent to

$$a \approx (a_N a_0)^{1/2} \quad (2)$$

in the deep-MOND regime, where  $a_N$  is the acceleration derived from Newtonian dynamics. It is just this simple relation that works remarkably well in reproducing the dynamics of a variety of galaxies with quite different morphologies and luminosities.

The modified dynamics can be interpreted as a modification of inertia, which, if applied properly, may solve a lot of puzzles faced with modern physics. Modification of inertia is not a new idea since the time of Mach who challenged Newton's idea about inertia. Since the theorization of Unruh radiation (Unruh 1976) and the discovery of Casimir effect (Casimir 1948), their relation to inertia and gravity has frequently been speculated by several authors (e.g. Puthoff 1989; Haisch, Rueda & Puthoff 1994). Though Casimir effect is a reality both theoretically and experimentally, what does it mean for and how to apply it to cosmology remains a subject of debate. Puthoff's gravity, based on Casimir effect, has the flaw of missing experimental support. As for Unruh radiation, it is not clear what this radiation means for cosmology and whether it is related to inertia.

In this paper, we propose a new approach, based on some general speculations, to modification of inertia. By this modification of inertia, (2) is successfully reproduced. Lagrangian formulae are developed and its indications are discussed then.

## 2 ASSUMPTIONS AND RESULTS

### 2.1 Assumptions

In this section, three assumptions are proposed: 1) Gravitational mass is conserved; 2) Inverse-square law is applicable at large distance; 3) Inertial mass depends on external fields.

If we make a comparison between gravity and electromagnetic interaction, we immediately realise that gravitational mass is analogous to electric charge. It is the electric charge, as a source, who produces electric field. Electric charge cannot be created and destroyed, it is just a being. Yang-Mills gauge theory, the foundation for standard model in particle physics, is based on the conservation of charge. As a source of gravitational field, it is unphysical that gravitational mass is not conserved.

In electromagnetic field, the inverse-square law is directly related to the zero-mass-ness of photons. Coulomb's law has been tested from  $\sim 2 \times 10^{10}$  m down to  $10^{-18}$  m, a magnitude span of 28 orders (Adelberger, Heckel & Nelson 2003). The astronomical tests of Newton's gravity has been mainly confined within solar system. The most accurate astronomical tests are lunar-laser-ranging studies of the lunar orbit, which do not show a deviation of gravity from Newton's law. On galactic scale, a Yukawa-like gravity has been proposed by many authors (see, e.g. Sanders & McGaugh 2002, and references therein) to account for the mass discrepancy in galaxies. However, Milgrom (1983) pointed out that this is inconsistent with the empirical Tully-Fisher law (Tully & Fisher 1977). This line of arguments indicate that gravity is not Yukawa-like from submillimeter scale to at least galactic scale. Large scale structure of the Universe favors a Newton's gravity law even on cosmic scale. Therefore it is quite safe to assume that inverse-square law is accurate on the scale of our interest in this paper.

The equivalence of gravitational mass to inertial mass, upon which general relativity is based, has been tested experimentally with very high accuracy (Will 2009). But these tests are confined within solar system, no direct test is available on the galactic scale. Unlike the gravitational mass, inertial mass does not associate with any physical field. Physically, because of the association with gravitational field, gravitational mass should be conserved, but this is not necessarily the case for inertial mass. Inertia is one object's ability to keep its original state of motion. However, how does one object know its original motion state if no reference is available?

Mach speculated that inertia is the result of the object's motion relative to the mean mass distribution of the Universe as a whole. In other words, inertia is meaningless if no mass is there other than the object itself. Let us consider another situation: If the mass is distributed uniformly throughout the Universe but one distinct object. Whatever the state of motion of this distinct object, the state of the Universe will keep the same. This is indicative of the dependence of inertia on the mass distribution of the Universe.

It has been long that Milgrom found that external field plays a role on the internal dynamics of open clusters where the internal field is well below the critical acceleration  $a_0$  and therefore should be in deep-MOND regime. However, the dynamics of these systems does not show any evidence of dark matter. Milgrom realised that external field, if significantly above the transitional acceleration between Newtonian dynamics and MOND, could play a role and make the dynamics of these open clusters Newtonian. In addition, according to Mach, gravitational field, being the result of the mass distribution of the Universe, therefore endows the object with inertia. This is slightly different from the original Mach's Principle. Other than simultaneously depending on the mass distribution of the whole Universe, here we speculate that inertial mass depends on the local gravitational field. This speculation is based both on Milgrom's realisation and on the present day basic belief that there cannot exist action-on-distance.

The above lines of argument indicate that if no external gravitational field exists, the object could be in random states of motion, i.e. the inertial mass is zero. With the increase of external field, inertial mass increases accordingly. The climbing-up of inertia, however, does not continue infinitely. Having the external field, i.e. the gradient of the potential, as a reference, the object has a sense of its past motion state. The stronger the external field, the more sense it has about its original motion state. But if the external field is strong enough, increasing the field's strength will not increase its sense of past motion state because it just has enough "information" about its original state of motion. As a result, we set the inertial mass  $m_i$  in the strong field limit to be its gravitational mass  $m_g$ , as we know from the dynamics of solar system.

However, in what a way the external gravitational field tells the moving particle of its past motion state? The only way that this can be

achieved is by the interaction of the particle's historical trajectory with the particle itself. That is to say, some present-day unrecognised field (referred to as S field hereafter) is released when the particle is in motion. If the S field is freely moving without any interruption, it will never interact with the particle who released it. In this case, the particle does not know its past motion state and is therefore inertialess. But if the S field it released is interrupted, the particle will acquire inertia because of its interaction with its prior history.

This argument seems still to be somewhat speculative. Let us now discuss a real situation where an inertialess object could naturally acquire inertia as a result of its interaction with its prior history. It is well known that crystal is never perfect and full of varying kinds of defects, e.g. line defects and point defects. Line defects are frequently referred to as cracks in fracture mechanics. The motion of a crack is determined by the stress field in the material under consideration. Based on a linear approximation, the classic equation of motion for a crack is beautifully described by the so-called linear elastic fracture mechanics (LEFM) (Freund 1998). LEFM predicts that cracks are inertialess if the medium is unbounded, which has been experimentally verified by Goldman, Livne, & Fineberg (2010).

On the other hand, if the medium is finite in size, Goldman et al. show that the motion of a crack is influenced by the boundaries of the medium so as to acquire inertia. This is possible because when a crack moves, the elastic waves surrounding the crack were reflected by the boundaries and have an influence on its motion. In other words, a moving crack interacts with its prior history. Goldman et al. further demonstrate that the inertia of a crack increases with crack velocity and becomes effectively infinite as the crack's velocity approaches a limiting velocity,  $c_R$ , viz. the speed of Rayleigh wave in this medium. This increase of inertia parallels the mass divergence of particles in special relativity. As a result, the speed of the proposed S field should be equal to the speed of light.

Then how a particle acquire inertia with the existence of external gravitational field? If we treat gravitational field as "boundaries" of the medium as for the case of crack, then a particle will acquire inertia in a similar way. The S field surrounding the moving particle will be reflected by gravitational field so as to influence the motion of the particle. If gravitational field vanishes, the particle will be inertialess because the particle is moving in a unbounded "medium". The stronger the gravitational field, the more the S field is reflected so that the particle's inertia is larger. When the gravitational field is strong enough that all S field is reflected, the particle's inertia will not increase and stays constant.

What happens if the gravitational field vanishes and particles are therefore inertialess? Do they move at a velocity which is larger than the speed of light? As we said before, the S field should be at a velocity that is equal to the speed of light. When the inertialess particles' velocity approaches the speed of light, they will be strongly interacted by the S field they just transmitted so that they acquire inertia. In this limit the particles all behave like photons. Therefore we conclude that the speed of light is still the limiting speed.

It should be stressed that inertia is only indirectly dependent on gravitational field. What gives a particle inertia is the interaction of the particle with the S field it transmitted previously. This point should be borne in mind when we derive gravitational field equation in Section 2.3.

By these three assumptions, we shall derive the MOND relation (2) in next section.

## 2.2 Results

Now we consider one particle's motion under the gravitational interaction of a massive object at large separation. The particle moves inward by converting potential energy to kinetic energy:  $\mathbf{F} = d\mathbf{p}/dt = (dm_I/dt)\mathbf{v} + m_I(d\mathbf{v}/dt)$ , where  $\mathbf{p} = m_I\mathbf{v}$  and  $m_I$ , of course, is its inertial mass, as usual. Because  $\mathbf{F} = -GMm_g\mathbf{r}/r^3$ , where  $M$  is the gravitational mass of the massive object, we get

$$m_I\mathbf{a} + \frac{dm_I}{dt}\mathbf{v} = -\frac{GMm_g}{r^3}\mathbf{r}. \quad (3)$$

For radial motion, we have

$$m_I vv' + m_I' v^2 = -\frac{GMm_g}{r^2}, \quad (4)$$

here the primes denote derivatives relative to the distance  $r$ . Substituting  $u$  for  $1/r$ , the above equation is reduced to

$$m_I v \frac{dv}{du} + \frac{dm_I}{du} v^2 = GMm_g. \quad (5)$$

To move further on, we have to figure out, under the guide of the third assumption proposed in above section, how  $m_I$  is shaped by external field. In general, based on the assumption that  $m_I \propto m_g$ , an inertial mass function of the form

$$m_I = \nu \left( \frac{g_N}{a_I} \right) m_g \quad (6)$$

is expected, where  $\nu(x)$  is a function of  $g_N/a_I$  only. As is well known,  $m_I$  is a constant in the strong field case, and by the third assumption,  $m_I \rightarrow 0$  in the weak-field extreme. We are interested in the weak-field extreme case. A plausible assumption for the weak field case is  $m_I \propto (g_N)^\alpha m_g$ , or

$$m_I(g_N \ll a_I) = \left(\frac{g_N}{a_I}\right)^\alpha m_g, \quad (7)$$

where  $g_N = GM/r^2 = GMu^2$ , the Newtonian gravitational field. Here we introduce a new constant,  $a_I$ , which is related to Milgrom's constant  $a_0$ , as can be seen in the next section. In order to smoothly bridge the strong field case and weak field case, we expect  $0 < \alpha < 1$ , which means  $m_I$  increases rapidly when  $g_N \ll a_I$  but ceases to climb up when  $g_N \simeq a_I$ . The index  $\alpha$  has to be fixed phenomenologically. As can be easily checked, if we set  $\alpha = 1/2$ , the desired MOND behaviour is recovered, i.e.

$$m_I(g_N \ll a_I) = \left(\frac{g_N}{a_I}\right)^{1/2} m_g, \quad (8)$$

where

$$a_I = v_0^4/GM, \quad (9)$$

and  $v_0$  the particle's asymptotic velocity. Because  $\nu(x) \approx 1$  when  $x \gg 1$ , a simple but quite plausible assumption is

$$\nu(x) = \left(\frac{x}{1+x}\right)^{1/2} \quad (10)$$

for all  $x$ . It should be stressed that, in this model,  $m_I/m_g$  is influenced only by external fields, not by any other factors, including the particle's dynamical quantities, e.g. its velocity. That is to say,  $m_I$  is a true scalar. Because of this behaviour of inertial mass, the particle's Lagrangian can be expressed as equation (13).

If the particle is on circular orbits, therefore a constant inertial mass, the equation of motion, by (3), is reduced to

$$m_g g_N = m_I a,$$

which reads

$$a = (g_N a_I)^{1/2},$$

i.e. the recovery of equation (2) if  $a_I = a_0$  is recognised. This equation, however, cannot be applied to any other orbits other than circular ones due to the variation of the particle's inertia. In general, equation (3) should be used. This indicates that the original MOND prescription is most suitable to describe the dynamics of spiral galaxies where internal motion is almost perfectly circular.

Now let us consider how to escape the gravity of a massive object. Substituting equation (8) into equation (5) we find, for a radial motion,

$$uv \frac{dv}{du} + v^2 = v_0^2. \quad (11)$$

This equation tells us that if the particle is moving away from the massive object and has a velocity  $v = v_0$ , then  $dv/du = 0$  and the particle keeps moving at a constant velocity. Therefore the particle is able to escape the gravitational pull of a massive object by itself.

If  $v < v_0$ , then  $dv/du > 0$  and the particle decelerates until reach a maximal distance. Of particular interest is the case of  $v > v_0$ , where  $dv/du < 0$  and the particle keeps accelerating and the acceleration is increasing. It is straightforward to check that its velocity satisfies the following equation

$$(v^2 - v_0^2)^{1/2} = Ar.$$

Obviously, if no other field's disturbance, the velocity will increase infinitely so that  $v \gg v_0$  and

$$v = Ar. \quad (12)$$

This is the direct result of the decreasing inertia while the particle moves away from the gravitational field. This equation is formally same as Hubble's law. But please bear in mind that Hubble's law describes velocity field, equation (12), on the other hand, describes one particular particle's velocity change with distance. we shall defer the discussion of this equation to Section 2.6.

It is clear that, since the particle's mass is dependent on the strength of gravity, gravity's apparent effect is not always attractive, but takes on different aspects dependent on the particle's motion state. If the particle is bound to the massive object, the gravity's apparent effect is attraction. On the other hand, if the particle is in an unbound state, the gravity's effect is repulsion, as indicated by (12), the particle's velocity is not decreasing but increasing as it moves away from the gravitational field.

### 2.3 Lagrangian Formalism

In above section we just assume that inertial mass depends on the external field and then extend Newtonian dynamics in a minimum manner. To formulate a self-consistent theory we need to develop a set of Lagrangian formulae. As usual, we write down the Lagrangian:

$$L = \frac{1}{2}m_I v^2 - m_g \phi, \quad (13)$$

where  $\phi$  is the scalar gravitational field that is determined by Poisson equation  $\nabla^2 \phi = 4\pi G \rho$  with  $\rho$  the gravitational mass density<sup>1</sup>. The Euler-Lagrange equations of motion

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{x}_i} \right) - \frac{\partial L}{\partial x_i} = 0$$

give

$$\frac{d\mathbf{p}}{dt} = -m_g \nabla \phi + \frac{1}{2} v^2 \nabla m_I. \quad (14)$$

Compared with equation (3) we find that equation (14) contains an additional term  $\frac{1}{2} v^2 \nabla m_I$ , stemming from the fact that inertial mass depends on external field, so is a function of position. It turns out that this term plays a key rule in accounting for the dynamics of dwarfs, spiral galaxies and clusters in a consistent and self-contained way. Applying  $\mathbf{p} = m_I \mathbf{v}$  to the left hand side of equation (14) gives:

$$m_I \mathbf{a} = m_g \mathbf{g}_N + \frac{1}{2} v^2 \nabla m_I - (\nabla m_I \cdot \mathbf{v}) \mathbf{v}. \quad (15)$$

For the deep-MOND case, substituting equation (8) into above equation yields

$$\mathbf{a} = \left( \frac{a_I}{g_N} \right)^{1/2} \mathbf{g}_N + \frac{1}{4} \frac{v^2}{g_N} \nabla g_N - \frac{1}{2} \frac{\nabla g_N \cdot \mathbf{v}}{g_N} \mathbf{v}. \quad (16)$$

This equation is the general motion equation for the deep-MOND regime, including the non-spherically symmetric systems. It is apparent from equation (15) that in a modified inertia theory, unlike modified gravity, the acceleration experienced by particle depends not only on position, but on the velocity as well. This seems a drawback of this kind of theory, but on the other hand this could be an unparalleled advantage (see, e.g., Milgrom 2002).

As above, let us first consider the circular motion. In this case the above equation, after applying equation (8), reduces to:

$$a = (a_I g_N)^{1/2} \left( 1 + \frac{1}{2} \frac{v^2}{\sqrt{G M a_I}} \right). \quad (17)$$

From this equation we can easily find the stable circular velocity

$$v_c = (4 G M a_I)^{1/4}. \quad (18)$$

However, only the sufficiently virialised systems can attain to this high circular velocity. For those systems less virialised, the objects on the outskirts of the systems will follow quasi-circular orbits and gradually wind up during the inward migration. This indicates that Milgrom's constant is actually slowly varying according to

$$a_0 = a_I \left( 1 + \frac{v^2}{v_c^2} \right)^2, \quad (19)$$

that is to say,  $a_0$  is varying in a narrow range  $a_I \leq a_0 \leq 4a_I$  and older systems tend to have higher values of  $a_0$ .

To find the value of  $a_I$ , an easy but reliable way is to carefully select a large enough sample of well studied galaxies so that the scatter of  $a_0$  is moderate and MOND works quite well for this sample. we select the galaxies from Begeman, Broeils & Sanders (1991), Sanders (1996) and Sanders & Verheijen (1998). The reasons for selecting these galaxies are three-fold. First, these galaxies are among the highest quality of observational data and it was demonstrated that MOND can account for the data with a relatively high precision. Secondly, these galaxies are all spiral galaxies for which MOND is most successful. In addition, as indicated above,  $a_0$  evolves with galaxies. To reduce the scatter of  $a_0$ , we should select galaxies with comparable properties. Finally, these three samples contain several galaxies, e.g. NGC 2841, that are quite controversial within the context of MOND. It is quite desirable if a varying  $a_0$  can settle down these issues. The resulting sample contains 63 spiral galaxies, but 9 galaxies from Sanders & Verheijen (1998) are eliminated from the list adopted to calculate  $a_0$  because of the reason presented below.

<sup>1</sup> As stated above, when we vary  $\phi$ , we should get Poisson equation because  $m_I$  is not directly dependent on gravitational field.

In Table 1 the calculated values of  $a_0$  are listed for every galaxy in the sample.  $a_I$  is found by requiring the average value of  $a_0 = 1.21 \times 10^{-8} \text{ cm s}^{-2}$ :

$$a_I = 0.667 \times 10^{-8} \text{ cm s}^{-2}. \quad (20)$$

Several points should be mentioned. The accelerations at the last measured points of the rotation curves ( $a_{\text{imp}}$ ) of NGC 3949, NGC 3953, NGC 4085, UGC 6973 are in the Newtonian regime and therefore eliminated from the calculation of  $a_I$ . In light of the found value of  $a_I$ , we expect that the galaxies with  $a_{\text{imp}}$  comparable to or larger than  $a_I$  will be quite dynamically different from other galaxies, these galaxies are NGC 3877, NGC 3972, NGC 4051, NGC 4217, NGC 4389, with  $a_{\text{imp}} > 0.7 \times 10^{-8} \text{ cm s}^{-2}$ . As a result, these five galaxies are eliminated from the calculation of  $a_I$ . But it should be mentioned that, if these five galaxies are included in the galaxies responsible for calculating  $a_I$ ,  $a_I$  would be slightly smaller. By this selection criterion, the resulting scatter of  $a_0$ ,  $\pm 0.30$ , around the mean value is moderate, as desired.

## 2.4 Comments on Selected Individual Galaxies

A quick glance of the result for  $a_0$  suggests that large and high surface brightness galaxies tend to have a high  $a_0$ . Several comments are presented below for those values significantly deviated from the standard value:

*NGC 2841.* – This is the most controversial spiral galaxy for MOND. Begeman, Broeils & Sanders (1991) found that, if a Hubble distance (9.5 Mpc) was used, MOND obviously fails to account for the rotation curve. On the other hand, if the Tully-Fisher distance was used, MOND is in well agreement with the observed data. However, the Tully-Fisher distance is twice as the Hubble distance, a seemingly unacceptable result. Subsequently, Bottema et al. (2002) found a distance of 14.1 Mpc to NGC 2841 based on Cepheid method. This distance goes half way between the Hubble distance and Tully-Fisher distance. As a result, the situation is alleviated somewhat, but the predicted curve still deviates systematically from the observed data. Table 1 gives this galaxy's  $a_0$  a value of  $1.48 \times 10^{-8} \text{ cm s}^{-2}$ , the largest in this sample if not include the values not adopted for calculating  $a_I$ , which is more than 22% larger than the standard value. This value will further alleviate the situation and make MOND in principle compatible with the data. But it should be pointed out that even with this high value, the data still can not be comfortably matched by the prediction. Here is a caveat. We find that, for this galaxy,  $a_{\text{imp}} = 0.66 \times 10^{-8} \text{ cm s}^{-2} \sim a_I$ . Because  $a_I$  is the transition acceleration beyond which the modified inertia transits to Newtonian inertia, we expect the value  $a_0$  calculated in this way is only qualitatively correct. If otherwise  $a_{\text{imp}} \ll a_I$ , the value listed in Table 1 should be exact. But NGC 2841 evolved beyond this stage because it has a much higher  $a_{\text{imp}}$ . Quantitatively, a value of  $1.87 \times 10^{-8} \text{ cm s}^{-2}$  for  $a_0$  will do the work if the Cepheid distance is used. If we adopt equation (10) as the inertial mass dependence on external gravitational field, we find a value of  $1.86 \times 10^{-8} \text{ cm s}^{-2}$  ( $\sim 2.77a_I$ ) for  $a_0$ , which is just what is needed for this galaxy to bring MOND prediction in accordance with the observation. As in the general case, when  $a_{\text{imp}} \sim a_I$ , the actual  $a_0$  depends on the ratio of rotation velocity to its stable circular velocity. When rotation velocity approaches its stable circular velocity,  $a_0$  will have a value of  $2.45 \times 10^{-8} \text{ cm s}^{-2}$  ( $\sim 3.66a_I$ ). This implies  $2.77a_I \leq a_0 \leq 3.66a_I$  when  $a_{\text{imp}} \sim a_I$ . We find that the maximum value ( $3.66a_I$ ) for  $a_0$  when  $a_{\text{imp}}$  is in the vicinity of  $a_I$  is less than the corresponding value ( $4a_I$ ) when  $a_{\text{imp}} \ll a_I$ , indicating that the Newtonian dynamics begin to take over when acceleration enters the Newtonian regime. The oscillating rotation curve, rather than a perfect flat rotation curve, of this galaxy on the outskirts may have some implication of this subtlety.

*DDO 154.* – A value of  $0.9 \times 10^{-8} \text{ cm s}^{-2}$ , significantly less than the average value, is given for this dwarf and gas-rich galaxy. Nonetheless, this low value can be largely attributed to the low rotation velocity at the last measured points. This is a quite controversial case for MOND. Milgrom & Braun (1988) demonstrated that this galaxy is among the most acute test of MOND. Shortly, Lake (1989), however, showed that a substantially small value of  $a_0$  is necessary to account for the observed curve. Milgrom (1991) disputed Lake's conclusion, arguing that a different distance to this galaxy be more reliable. But if the ideas presented in this paper are correct, a small value of  $a_0$  is inevitable because of the declining nature of the rotation curve at the outermost points. The internal acceleration of DDO 154 is everywhere less than  $a_0$  so that MOND should play a dominant role. But MOND cannot account for the declining nature of the rotation curve. In the spirit of this paper, however, the declining nature of gas-rich dwarf galaxies can be naturally accounted for. The main point is that gas-rich galaxy is newly formed object with a less relaxed outer part and a fairly relaxed inner part. Consequently, the inner part should have a higher value of  $a_0$  than its outer part. This indicates that  $a_0$  is varying not only from galaxy to galaxy, but also within galaxies. For sufficiently relaxed galaxies,  $a_0$  could be treated as a constant without loss of accuracy, but for gas-rich galaxies, the variation of  $a_0$  is maximised.

*NGC 3893.* – Table 1 gives  $a_0$  a value of  $1.47 \times 10^{-8} \text{ cm s}^{-2}$ . This high value is largely the result of the high  $a_{\text{imp}}$ . But because this galaxy has a disturbed velocity field, the result should not be taken seriously.

*NGC 3992.* – Table 1 gives  $a_0$  a value of  $1.48 \times 10^{-8} \text{ cm s}^{-2}$  or  $1.86 \times 10^{-8} \text{ cm s}^{-2}$  if equation (10) is used, an apparently not necessary adjustment for the standard value if the good agreement between predicted and observed rotation curve is noticed. However, as mentioned by Sanders & Verheijen (1998), the fitted value of mass-to-light ratio of the stellar disk is unusually large compared with other values in that sample. If, however, an  $a_0$  with above value is used, the near-infrared mass-to-light ratio will be reduced by a factor of 1.22 or 1.54, a value compatible to other galaxies in the sample.

*NGC 4010.* – Table 1 gives  $a_0$  a value of  $1.44 \times 10^{-8} \text{ cm s}^{-2}$  and even higher if equation (10) is used. This high value of  $a_0$  seems unexpected for such a low surface brightness galaxy. However, this galaxy has a very extended disk and the scale length is much larger compared with other galaxies in its mother cluster (Tully & Verheijen 1997). As a result, this galaxy has a much larger internal acceleration, so is  $a_0$ . This galaxy is an example that  $a_0$  does not actually directly depend on surface brightness. Since young galaxies with outstanding

star formation activities tend to have a high surface brightness even if its internal acceleration is low. On the other hand, some old galaxies devoid of star formation tend to have a low surface brightness because of their old star population. In addition, we find that the rotation curve of this galaxy begins to decline at large radii, another example that  $a_0$  is varying within one galaxy.

Recently Swaters, Sanders & McGaugh (2010) study a sample of 27 dwarf and low surface brightness galaxies within the framework of MOND and find that there are some systematics that cannot be reconciled easily: MOND curve predicts higher rotation velocities in the outer regions and lower in the central regions for low surface brightness dwarfs; higher rotation velocities in the central regions and lower in the outer regions for high surface brightness galaxies (UGC 5721, UGC 7323, UGC 7399, UGC 7603, UGC 8490). This intriguing phenomenon forces Swaters et al. to try a fit with  $a_0$  as a free parameter. By allowing  $a_0$  vary, Swaters et al. find a trend that lower surface brightness galaxies tend to have lower  $a_0$ . This is just what equation (19) expected if lower surface brightness galaxies tend to be less relaxed. If equation (19) is applied to this sample of systems, an average value of  $1.06 \times 10^{-8} \text{ cm s}^{-2}$  is found, which is significantly lower than the commonly used value. Swaters, Sanders & McGaugh (2010), however, find an even lower average value of  $0.7 \times 10^{-8} \text{ cm s}^{-2}$  for  $a_0$  in this sample, a value that is just above  $a_I$ . Though the calculated value of  $a_0$  is significantly lower than the commonly used value, it is still significantly higher than the fitted average value in this sample. Furthermore, the fitted values for the galaxies in this sample have an enormously large scatter that is completely out of the reach for (19) to account for. This enormous scatter, on the one hand, is a consequence of the internal dynamics of the systems; on the other hand, is a consequence of observational uncertainties. If such a large scatter of  $a_0$  is necessary for this sample, then it will be a challenge for the ideas presented in this paper.

## 2.5 Dynamical Mass of Clusters of Galaxies

It has been quite controversial with the applicability of MOND to clusters. The & White (1988) first noted that MOND is inconsistent with the dynamics of Coma cluster unless the MOND acceleration parameter  $a_0$  is 4 times larger than the value inferred from spiral galaxy rotation curves. Sanders (1994), based on a small sample of X-ray-emitting clusters, concluded that MOND mass is consistent with the detected gas mass. Subsequently, based on a much larger sample of X-ray-emitting clusters, Sanders found that MOND over-predicted the mass by a factor of 2 than the detected gas mass (Sanders 1999). In light of this finding, the mass over-prediction was already implied by Sanders (1994) though with a less statistical significance.

This mass over-prediction flaws MOND since MOND was particularly speculated to eliminate non-baryonic dark matter. To remedy MOND, Sanders (2003) suggested that neutrinos, with mass of 2 eV, is responsible for the mass discrepancy in rich clusters. This mass of neutrinos, having a negligible effect on the galaxy scale, could have a significant dynamical effect on cluster scale. Unfortunately, Angus, Famaey & Buote (2008) showed that neutrinos are incompatible with the observed mass distribution within X-ray bright groups and clusters. Consequently, Angus et al. proposed that sterile neutrinos would close the cluster mass problem. This conclusion contradicted the finding by Milgrom (1998) who found that MOND is compatible with galaxy groups. Unfortunately, by studying the gravitational lensing of clusters, Natarajan & Zhao (2008) conclude that even sterile neutrinos are far from enough to close the mass budget of clusters.

However, if the inertia is allowed to vary, as presented above, no dark matter is needed. Of the 93 clusters studied by Sanders (1999), the internal accelerations are of  $\sim 0.6 \times 10^{-8} \text{ cm s}^{-2}$ , that is, in the vicinity of  $a_I$ . Section 2.4 shows that for systems with internal accelerations comparable to  $a_I$ , a much higher effective  $a_0$  should be used. It can be checked that for these 93 clusters,  $a_0$  approaches the maximum value  $3.66a_I$  ( $2.02 \times 1.21 \times 10^{-8} \text{ cm s}^{-2}$ ). This high value is just enough to eliminate the mass discrepancy in bright clusters. In this respect, we say that the dynamics of bright clusters are quite similar to that of NGC 2841. This also implies that not all clusters would have the mass discrepancy problem within the framework of MOND. Those clusters with much lower internal accelerations will have similar dynamics as spiral galaxies.

But the mass discrepancy problem is not completely closed because in some cases MOND over-predicts masses by a factor of 3 or even higher. If this phenomenon is ubiquitous and no conventional interpretations, e.g. dark baryons or neutrinos, are reliable, then there really exists a dark matter problem for MOND.

## 2.6 Unbound Systems

After a discussion of circular motion, let us now turn to radial motion. In this case equation (15) reduces to

$$a = (a_I g_N)^{1/2} \left( 1 - \frac{1}{2} \frac{v^2}{\sqrt{GM a_I}} \right).$$

We find the conclusion is not changed much for radial motion by an application of Lagrangian formula. The only difference is a change of the asymptotic velocity, defined by equation (18) other than the value  $v_0$  determined by equation (9).

Equation (12) reminds us of the Hubble's law. But does it really imply Hubble's law. By its low magnitude ( $H_0 = 72 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ) compared to the typical galaxies' asymptotic rotation velocities ( $\simeq 200 \text{ km s}^{-1}$ ) and their typical sizes ( $\lesssim 100 \text{ kpc}$ ), if it is really Hubble's law, we expect that Hubble's law is not the result of mass inhomogeneity on the galactic scale. On contrary, Hubble flow is a result of mass inhomogeneity on cosmic scale. Obviously, if the mass distribution is completely homogeneous, galaxies would not feel the existence of an external gravitational field, resulting in zero inertial masses and therefore random motions. But actually every galaxy is embedded in an

ubiquitous gravitational field. A fluctuation of mass density on cosmic scale gives rise to the small gravity on large scale, those cosmic objects unbound by this inhomogeneity of mass distribution acceleratingly expand. If  $\delta M$  is the mass inhomogeneity caused by a density perturbation  $\Delta = \delta\rho/\rho$  on the cosmic scale, we expect the Hubble recession velocity exceeds  $\delta v_0 = (4G\delta M a_I)^{1/4}$  so that an accelerating expansion is guaranteed. This indicates that the accelerating expansion of the Universe can be most readily observed on scales that the clustering of galaxies becomes negligible. The two-point correlation function of galaxies shows that these scales are  $r \simeq 5h^{-1}$  Mpc. So the accelerating expansion of the Universe should be free of peculiar contamination beyond these scales. The accelerating expansion of the Universe seems to imply an increase of the universe energy, or in modern parlance, the existence of dark energy. But above analysis shows that the accelerating expansion of Universe does not violate conservation of energy because equation (12) is derived based on the conservation of energy. All difference is the dependence of inertia on gravity. Although galaxies' velocities are becoming larger and larger, their inertial masses are dropping dramatically, resulting in the conservation of total energy.

Although we recover MOND's relation (2) in the case of circular orbits, the ideas presented above are quite different from that of MOND. In MOND, gravity's effect is always attractive, the moving particles in the gravitational field are all feeling the same acceleration as long as they locate at the same place in the field. In fact MOND, including the relativistic MOND theory proposed by Bekenstein, complies with weak equivalence principle. This is however not the case for the ideas presented here. Just as we already find, gravity is attractive in the case of bound state, and repulsive in an unbound configuration. Even for particles in the same type of state, say bound state, their accelerations could be quite different because of their different radial velocities, which determine at what rate the moving particles' inertial masses change. As a consequence, all particles moving from large distance towards the massive object tend to converge to the same velocity, i.e. the asymptotic velocity  $v_0 = (4GMa_I)^{1/4}$ . If the particles' velocities are less than  $v_0$ , the attraction effect of the gravity increases their velocities. On the other hand, if the particles' velocities are larger than  $v_0$ , the repulsion effect of the gravity decrease their velocities. Only in the Newtonian regime is the gravity always attractive.

In addition, since the gravitational potential is not infinite, opposed to the original MOND, the moving body can escape the gravitational pull easily without resorting to other nearby massive galaxies' assist as described by Famaey, Bruneton & Zhao (2007). This is yet another advantage of the changing inertia proposal over the traditional MOND. In many cases, by finely tuning the parameters, the predictions of CDM model are barely discernible from that of MOND. But globular clusters are outliers, which are believed contain no dark matter. Consequently Baumgardt, Grebel & Kroupa (2005) suggested a test of gravitational theories by studying velocity dispersion of globular clusters. Recently these studies are carried out by several groups, finding an inconsistency between MOND and observations (Moffat & Toth 2008; Haghi et al. 2009; Sollima & Nipoti 2010). The problem is that MOND systematically overpredicts the velocity dispersions. This can be understood as follows: Because of the infinite escape velocity intrinsic in MOND, at any given radius the velocity distribution should be large. This is no longer the case for the changing inertia proposal since at any radius the escape velocity is finite.

### 3 DISCUSSION

#### 3.1 Indications for structure formation and evolution

The cosmology and structure formation in the context of MOND were first studied by Felten (1984) and then extended by Sanders (1998), Sanders (2001) and Stachniewicz & Kutschera (2001). The main findings of these studies are that MOND would boost the structure formation even with a very low matter density. van den Bosch & Dalcanton (2000) semianalytically studied the formation of disk galaxies and found that dark matter and MOND models are indistinguishable. Stachniewicz & Kutschera (2005) found that the dark ages would end at  $z \sim 30$ , in good agreement with the WMAP results. More recent studies (Sanders 2008; Malekjani, Rahvar & Haghi 2009) of structure formation generally strengthen these early conclusions. When confronting MOND with the formation of galaxy clusters, however, Nusser & Pointecouteau (2006) found that the simulated MOND clusters are significantly denser than the observed ones. It means that MOND, as a gravitational field, is so strong that it over-evolves the structure on cluster scale.

If the ideas presented in this paper are correct, we find that the changing inertia postulate boosts the structure formation but the evolution is only moderately boosted. This is because when the particles begin to fall into the potential well of the density perturbation, thanks to their low initial velocities, therefore small variation of their inertial masses in unit time due to small variation of the potential well in which they are moving, the accelerations of the particles are almost the same as that predicted by the standard MOND. This indicates that the timescale of initial formation is barely different from that of standard MOND. But when the particles gain enough velocities, owing to the rapid variation of their inertial masses, their accelerations are quite small compared with the ones predicted by the standard MOND. Therefore the structure dynamical evolution is deferred. This effect will make the structure less dense than that predicted by MOND. van den Bosch & Dalcanton (2000) also found that stellar feedback is needed to reproduce the lack of high surface brightness dwarf galaxies within the context of MOND. However, the changing inertia proposal will defer the evolution of structure so that central surface brightness could be significantly lower.

This line of arguments also indicates that older objects in a system should be kinematically hotter than their younger counterparts. It is well known that there exists an age-velocity relation (AVR) in the disk of spiral galaxies (cf. Meusinger, Stecklum & Reimann 1991). The ideas presented in this paper indicate that there should be a rapid rise of the velocity dispersion of disk stars and then saturate when they are old enough. Unfortunately, the kinematical evolution of the disk is still controversial. Edvardsson et al. (1993) and Soubiran et al. (2008) found that there is a rapid rise of velocity dispersion for the first 2 Gyr and then saturate for older stars. Nordström et al. (2004) and Rocha-Pinto et al. (2004), on the other hand, found a steady increase in the velocity dispersion with age. Holmberg, Nordström & Andersen



(2007), with an improvement on  $T_{\text{eff}}$  and [Fe/H] calibrations for early F stars, found a steady increase in velocity dispersion with age and a tentative saturate at age 10 Gyr.

The age-velocity relation is usually attributed to disk heating. Several heating mechanisms, e.g. scattering by giant molecular clouds, spiral structures, and accretion, have been proposed to account for this relation. But it is growingly evident that these mechanisms are insufficient to account for the observations (cf. Rocha-Pinto et al. 2004). The ideas presented in this paper may shed new light on this issue because they will boost the velocity dispersion.

### 3.2 More words on inertia

When we argue for a dynamical origin of inertia, we mentioned cracks and the acquisition of their inertia as an interaction with their prior histories by introducing boundaries for the medium. Here we present evidence that why cracks are of particular interest. Cracks, the line defects in material, have their siblings in crystal, viz. dislocations and disclinations. Since the development of defect gauge field theory in condensed matter physics, it has been more and more clear that Einstein's gravitational theory is a special case of the gauge theory for defected crystal in the sense that general relativity is free of torsion (cf. Kleinert 1990). The geometrical study of crystal finds that a crystal filled with dislocations and disclinations has the same geometric properties as an affine space with torsion and curvature, respectively.

The Einstein curvature tensor,  $G_{\mu\nu}$ , in its three-dimensional version, is equivalent to the disclination density tensor,  $\Theta_{ij}$ . As a result, the Einstein field equation,  $G_{\mu\nu} = -8\pi GT_{\mu\nu}$ , indicates that the baryons are just disclinations in the "world crystal" with Plank length as the lattice constant (Kleinert 1990). Then why do we not observe a space with torsion? Torsion can only be a result because of the existence of dislocations. However, dislocation would make the space discontinuous and cannot exist macroscopically because it is unable to maintain stress equilibrium by itself. This is, however, not to say that dislocations do not exist microscopically. Actually the dislocation density,  $\alpha_{ij}$ , is related to disclination density by  $\partial_i \alpha_{ij} = -\varepsilon_{jkl} \Theta_{kl}$ . That is to say, several dislocations could end up as a disclination as seen by a distant observer. Therefore dislocations are akin to quarks in the way that dislocations and quarks are always confined within disclinations and baryons, respectively. If quarks are indeed dislocation defects in world crystal, then we should expect that space is not torsion free on the quark scale. Furthermore, the conservation laws for the angular momentum and energy-momentum density in general relativity are just the respective conservation laws for dislocation and disclination density in defected world crystal (Kleinert 1990).

This indicates that the argument for the way particles acquire inertia in a similar way in which a crack acquires inertia in a bounded medium is not just a trick but a facet of reality.

## 4 CONCLUSIONS

MOND has been well established as a serious alternative to the standard dark matter model for about three decades owing to its remarkable success in accounting for the dynamics of a variety of galaxies with quite different luminosities, morphologies. However, when confronting with dwarfs and clusters, MOND is controversial. There exists cumulated evidence that the acceleration parameter  $a_0$ , assumed to be universal, is varying in the sense that low surface brightness galaxies tend to have low  $a_0$ . For bright clusters, a factor of 2 mass over-prediction is well established. Again, this over-prediction can be accommodated by an  $a_0$  2 times larger than the usually adopted value.

With above observational evidence, a motivation to theoretically account for the phenomena is prompt. This paper postulates that the MOND phenomenology can be accounted for by three assumptions: 1) Gravitational mass is conserved; 2) Inverse-square law is applicable at large distance; 3) Inertial mass depends on external fields. The first two assumptions are quite general and should not find objection. The third assumption, given by equation (8) appropriate at the low acceleration regime, is key to reproduce the phenomenology of MOND. It is found that the inertia modified in this way exactly recover the formulae suitable to MOND in the circular motion case.

By a consideration of Lagrangian formulae, however, it is found that the usual relation (2) is replaced by (17). A comparison of equations (2) and (17) shows that  $a_0$ , a proposed universal constant, is actually varying in a narrow range,  $a_1 \leq a_0 \leq 4a_1$ , if the internal accelerations of the systems in question (or the external field) are much lower than  $a_1$ . A scrutiny of equations (15) and (10) indicates that the effective  $a_0$  not only depends on the orbital velocity but also on the external field. This varying  $a_0$  is just enough to eliminate the over-prediction factor of 2 found in bright clusters and the lower value in low surface brightness galaxies. Because of this nature of  $a_0$ ,  $a_0$  not only varies from system to system, but also varies within one galaxy. This phenomenon can readily account for the decline of rotation curves found for many low surface brightness galaxies.

Above statement suits only for circular motion. For radial motion, situation is quite different. It turns out that there exists a critical velocity,  $v_c$  defined by equation (18), for every galaxy. If the object in radial motion is moving slower than this critical velocity, gravity appears to be attractive, as usual. But for a motion faster than this critical velocity, gravity appears to be repulsive in the sense that the object begins to accelerate. This immediately reminds us of the accelerating expansion of the universe. By a careful inspection of the cosmic parameters, if the modified inertia is correct, it is realised that the accelerating expansion of the universe must be the result of inhomogeneity of the universe on cosmic scales.

In general, besides circular motion and radial motion, equations (15) and (10) should be used to account for the dynamics of systems in question. These two equations indicate that the traditional MOND prescription is only appropriate to describe circular motions. It is found that  $a_0$  is not a fundamental constant, its value is an indication of the system's age and relaxation index. In general, high surface brightness

objects tend to have high value of  $a_0$ . But this is not appropriate for all systems.  $a_0$  is actually determined by the orbital velocities and external gravitational fields. A more accurate value of  $a_I$  should be determined by these two equations. But owing to the success of MOND in spiral systems, the true value of  $a_I$  should not deviate far from the value given by equation (20).

## ACKNOWLEDGMENTS

I am grateful to Robert H. Sanders for his encouragement in this field. I also wishes to thank Moti Milgrom for his help during the year the ideas discussed here were conceived. I thank HongSheng Zhao for his critical reading of the manuscript and helpful comments.

## REFERENCES

- Adelberger E. G., Heckel B. R., Nelson A. E., 2003, *ARNPS*, 53, 77  
 Angus G. W., Famaey B., Buote D. A., 2008, *MNRAS*, 387, 1470  
 Angus G. W., Shan H. Y., Zhao H., Famaey B., 2007, *ApJ*, 654, L13  
 Baumgardt H., Grebel E. K., Kroupa P., 2005, *MNRAS*, 359, L1  
 Begeman K. G., Broeils A. H., Sanders R. H., 1991, *MNRAS*, 249, 523  
 Bekenstein J., Milgrom M., 1984, *ApJ*, 286, 7  
 Bekenstein J. D., 2004, *Phys. Rev. D*, 70, 083509  
 Bottema R., Pestana J. L. G., Rothberg B., Sanders R. H., 2002, *A&A*, 393, 453  
 Casimir H. B. G., 1948, *Proc. Kon. Nederland. Akad. Wtensch.*, B51, 793  
 Edvardsson B., Andersen J., Gustafsson B., Lambert D. L., Nissen P. E., Tomkin J., 1993, *A&A*, 275, 101  
 Famaey B., Bruneton J.-P., Zhao H., 2007, *MNRAS*, 377, L79  
 Felten J. E., 1984, *ApJ*, 286, 3  
 Freund L. B., 1998, *Dynamic Fracture Mechanics*, Cambridge University Press, Cambridge  
 Gentile G., Zhao H. S., Famaey B., 2008, *MNRAS*, 385, L68  
 Goldman T., Livne A., Fineberg J., 2010, *Phys. Rev. Lett.*, 104, 114301  
 Haghi H., Baumgardt H., Kroupa P., Grebel E. K., Hilker M., Jordi K., 2009, *MNRAS*, 395, 1549  
 Haisch B., Rueda A., Puthoff H. E., 1994, *Phys. Rev. A*, 49, 678  
 Holmberg J., Nordström B., Andersen J., 2007, *A&A*, 475, 519  
 Kleinert H., 1990, *Gauge Fields in Condensed Matter*, World Scientific  
 Lake G., 1989, *ApJ*, 345L, 17  
 Malekjani M., Rahvar S., Haghi H., 2009, *ApJ*, 694, 1220  
 Meusinger H., Stecklum B., Reimann H.-G., 1991, *A&A*, 245, 57  
 Milgrom M., 1983, *ApJ*, 270, 365  
 Milgrom M., 1991, *ApJ*, 367, 490  
 Milgrom M., 1998, *ApJ*, 496L, 89  
 Milgrom M., 2002, *New Astron. Rev.*, 46, 741  
 Milgrom M., Braun E., 1988, *ApJ*, 334, 130  
 Moffat J. W., Toth V. T., 2008, *ApJ*, 680, 1158  
 Natarajan P., Zhao H. S., 2008, *MNRAS*, 389, 250  
 Nordström B., Mayor M., Andersen J., Holmberg J., Pont F., Jørgensen B. R., Olsen E. H., Udry S., Mowlavi N., 2004, *A&A*, 418, 989  
 Nusser A., Pointecouteau E., 2006, *MNRAS*, 366, 969  
 Pointecouteau E., Silk J., 2005, *MNRAS*, 364, 654  
 Puthoff H. E., 1989, *Phys. Rev. A*, 39, 2333  
 Reyes R., Mandelbaum R., Seljak U., Baldauf T., Gunn J. E., Lombriser L., Smit R. E.h., 2010, *Nature*, 464, 256  
 Rocha-Pinto H. J., Flynn C., Scalo J., Hänenen J., Maciel W. J., Hensler G., 2004, *A&A*, 423, 517  
 Sanders R. H., 1994, *A&A*, 284L, 31  
 Sanders R. H., 1996, *ApJ*, 473, 117  
 Sanders R. H., 1998, *MNRAS*, 296, 1009  
 Sanders R. H., 1999, *ApJ*, 512L, 23  
 Sanders R. H., 2001, *ApJ*, 560, 1  
 Sanders R. H., 2003, *MNRAS*, 342, 901  
 Sanders R. H., 2008, *MNRAS*, 386, 1588  
 Sanders R. H., McGaugh S. S., 2002, *ARA&A*, 40, 263  
 Sanders R. H., Verheijen M. A. W., 1998, *ApJ*, 503, 97  
 Sollima A., Nipoti C., 2010, *MNRAS*, 401, 131

- Soubiran C., Bienaymé O., Mishenina T. V., Kovtyukh V. V., 2008, A&A, 480, 91
- Stachniewicz S., Kutschera M., 2001, Acta Physica Polonica B, 32, 3629
- Stachniewicz S., Kutschera M., 2005, MNRAS, 362, 89
- Swaters R. A., Sanders R. H., McGaugh S. S., 2010, ApJ, 718, 380
- The L. S., White S. D. M., 1988, AJ, 95, 1642
- Tully R. B., Fisher J. R., 1977, A&A, 54, 661
- Tully R. B., Verheijen M. A. W., 1997, ApJ, 484, 145
- Unruh W. G., 1976, Phys. Rev. D, 14, 870
- van den Bosch F. C., Dalcanton J. J., 2000, ApJ, 534, 146
- Will C. M., 2009, Space Sci. Rev., 148, 3

Table 1. The sample selected for the calculation of  $a_1$ . Calculated values of  $a_0$  are listed on column 2. Reference: (1) Begeman, Broeils & Sanders (1991); (2) Sanders (1996); (3) Sanders & Verheijen (1998). Nine galaxies from Sanders & Verheijen (1998), NGC 3877, NGC 3949, NGC 3953, NGC 3972, NGC 4051, NGC 4085, NGC 4217, NGC 4389, UGC 6973, are eliminated from the calculation of  $a_1$  because of the reason presented in text. The data for UGC 6446 are taken from Swaters, Sanders & McGaugh (2010) to utilise the most recent update, but the differences are small.

Galaxy	$a_0$ ( $10^{-8}$ cm s $^{-2}$ )	ref
NGC 2403	1.20	1
NGC 2841	1.48	1
NGC 2903	1.32	1
NGC 3109	1.01	1
NGC 3198	1.11	1
NGC 6503	1.06	1
NGC 7331	1.29	1
NGC 1560	1.12	1
UGC 2259	1.24	1
DDO 154	0.90	1
DDO 170	0.99	1
UGC 2885	1.28	2
UGC 5533	1.17	2
NGC 6674	1.16	2
NGC 5907	1.34	2
NGC 2998	1.20	2
NGC 801	1.12	2
NGC 5731	1.24	2
NGC 5033	1.24	2
NGC 3521	1.25	2
NGC 2683	1.31	2
NGC 6946	1.16	2
UGC 128	1.00	2
NGC 1003	0.98	2
NGC 247	1.22	2
M33	1.32	2
NGC 7793	1.36	2
NGC 300	1.09	2
NGC 5585	1.10	2
NGC 2915	1.06	2
NGC 55	1.15	2
IC 2574	1.05	2
DDO 168	1.14	2
NGC 3726	1.20	3
NGC 3769	1.02	3
NGC 3893	1.47	3
NGC 3917	1.33	3
NGC 3992	1.48	3
NGC 4010	1.44	3
NGC 4013	1.26	3
NGC 4088	1.38	3
NGC 4100	1.31	3
NGC 4138	1.31	3
NGC 4157	1.30	3
NGC 4183	1.11	3
UGC 6399	1.24	3
UGC 6446	1.05	3
UGC 6667	1.23	3
UGC 6818	1.17	3
UGC 6917	1.31	3
UGC 6923	1.44	3
UGC 6930	1.23	3
UGC 6983	1.15	3
UGC 7089	1.14	3